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PROPOSAL FOR FUZZY CONNECTIVES WITH A LEARNING FUNCTION USING THE STEEPEST DESCENT METHOD

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In conventional retrieval systems, retrieved results do not always satisfy user requests since only some operators for "and" and "or" in reasoning queries are defined. In this paper, we propose new fuzzy connectives which can express "and/or" operators that provide users with results satisfactory to their request. The fuzzy connectives are defined using a convex combination with a weight function of the parametric *t*-norm and *t*-conorm proposed by Schweizer. When user estimates for retrieved results are given, parameters of the fuzzy connectives are adjusted to minimize the sum of square errors between user estimates and the output of fuzzy connectives by the steepest descent method. The fuzzy connectives are called "fuzzy connectives with a learning function," which means any operators between the drastic product and drastic sum. They can represent a linear mean operator between a *t*-norm and *t*-conorm by using convex combination. We report the formulation of fuzzy connectives with a learning function and present the results of a retrieval experiment which demonstrates their usefulness.

Keywords: *fuzzy retrieval, fuzzy connectives, t-norm, t-conorm, mean operator, steepest descent method, learning function*

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1. INTRODUCTION

There have been many studies (see, e.g., [1–8]) on retrieval methods which treat human uncertainty using fuzzy theory, where fuzzy sets have represented the uncertainty of such words as "cheap" and "near," and such retrieval queries as "search for a hotel that is cheap and near the place I must go to conduct business." It would be most practical for retrieval queries to take a more natural form. Retrieval in this context includes seeking the correspondence of fuzzy sets for each piece of retrieval data, executing "and/or" operations, calculating the correspondence for retrieval queries, and outputting retrieval results in order of high to low correspondence. In conventional fuzzy retrieval systems, different users always obtain the same retrieval results, though they do not always satisfy the subjective user requests

One retrieval method was recently suggested in which the parameters of the retrieval algorithm are adjusted by employing user estimates of retrieval results as the degree of satisfaction of a retrieval request so as to increase satisfaction and yield results which satisfy the user. Ogawa et al. [9] proposed a document retrieval system which adjusts the membership values which express the relationship among the keywords of a document and retrieves the document which the user most wants. Maeda and Murakami [10,11] suggested a retrieval system which adjusts "and/or" operators within retrieval queries and retrieves results which satisfy the user; however, the retrieval relationship among document keywords must be set in advance so as to process the huge database required, the matrix must be squared, and the dimensionality of processible databases is limited. Zimmermann proposed fuzzy connectives which can express t-norms [12], t-conorms [12], and mean operators [13], which he extended and used in a later-developed retrieval system. Zimmermann fuzzy connectives [14] are multi-input fuzzy connectives which operators between the drastic product and drastic sum, including mean operators. Maeda and Murakami [10,11] also suggested a method which expresses operators to determine the weight of inputs and adjusts operator parameters to minimize the error between operator result and output data; however, Maeda fuzzy connectives can only express operators between the algebraic product and algebraic sum. It cannot express operators for the t-norm of a drastic product or the t-conorm of a drastic sum, etc.

We discuss adjustment of fuzzy connectives herein. In fuzzy retrieval systems, fuzzy sets with multi-order dimension are expressed by "and/or" operators among the fuzzy sets of each input dimension in retrieval queries, the membership value for each piece of data in the database is calculated, and the result is output in order of high to low. A retrieval query with fuzzy sets P_1 and P_2 is expressed as $P = P_1$ and P_2 (regarding "employee = high salary and age about 30" as the retrieval query). If the membership functions of P_1 and P_2 are expressed by $\mu_{P_1}(x_1)$ and $\mu_{P_2}(x_2)$ and operator "and" is represented by Ⓣ, the membership function of the retrieval query is written as

$$\mu_P(\boldsymbol{x}) = \mu_{P_1 \cap P_2}(x_1, x_2) = \mu_{P_1}(x_1) \textcircled{T} \mu_{P_2}(x_2).$$

The retrieval result is written as

$$\{\boldsymbol{x}^*\} = \{\boldsymbol{x} \mid \max \mu_P(\boldsymbol{x})\}.$$

To obtain $\boldsymbol{x}^*$ which satisfies a user request better, we adjust the membership functions rather than adjusting the fuzzy connectives. The shape of $\mu_{P_1}(x_1)$ and $\mu_{P_2}(x_1)$ are adjusted to make $\mu_P(\boldsymbol{x}^*)$ largest. A membership function with multi-order dimension $\mu_{P_1 \cap P_2}(x_1, x_2)$ is adjusted in this fashion. However, fuzzy sets with multi-order dimension $P_1 \cap P_2$ is expressed by using operator Ⓣ("and/or" operators). Therefore, even though the shape of $\mu_{P_1}(x_1)$ and $\mu_{P_2}(x_2)$ are adjusted, we cannot be sure that the membership values $\mu_{P_1 \cap P_2}(x_1^*, x_2^*)$ will be larger. We can conclude that it is more natural to express operators Ⓣ ("and/or" operators) that correspond to x_1 and x_2 and adjust the operator parameters to improve the satisfaction of user requirements than it is to adjust the membership functions.

We propose new fuzzy connectives which express "and/or" operators of retrieval queries that satisfy user requests and which are defined using a convex combination with a weight function of parametric the t-norm and t-conorm which was proposed by Schweizer [13]. We also propose a method where the parameters of fuzzy connectives are adjusted to minimize the sum of square error between user estimates and the output of fuzzy connectives by the steepest descent method. When user estimates of a retrieval result are given, the parameters of t-norm and t-conorm are adjusted to minimize the sum of square error between the user estimate and the fuzzy connective output. The fuzzy connectives we propose are called "fuzzy connectives with a learning function." Maeda fuzzy connectives express operators between the algebraic product and algebraic sum, but connectives with a learning function not only express operators between the drastic product and drastic sum but also represent linear mean operators between the t-norm and t-conorm.

We propose formulation of fuzzy connectives with a learning function, and present the results of a retrieval experiment where fuzzy connectives with a learning functions are employed to demonstrate their usefulness.

2. FUZZY CONNECTIVES

We call the t-norm, t-conorm, and mean operators [13] "fuzzy connectives." t-norm T is a function of $T(x_1, x_2)$: $[0,1] * [0,1] \to [0,1]$, and the following conditions must hold:

$$T(x,1) = x, \;\; T(x,0) = 0, \tag{1}$$

$$T(x_1, x_2) \leq T(y_1, y_2) \;\; (x_1 \leqq y_1, \, x_2 \leqq y_2), \tag{2}$$

$$T(x_1, x_2) = T(x_2, x_1), \tag{3}$$

$$T(x_1, T(x_2, x_3)) = T(T(x_1, x_2), x_3), \tag{4}$$

where (1) is the boundary condition, (2) is the monotonicity condition, (3) is the commutativity condition, and (4) is the associativity condition.

Dual function t-conorm S is obtained by the following formula:

$$S(x_1, x_2) = 1 - T(1 - x_1, 1 - x_2). \tag{5}$$

Like a t-norm, t-conorm S must also satisfy the above four conditions.

The following operators are typical operators of t-norms and t-conorms:

1) t-norm

$$\text{Logical product: } x_1 \wedge x_2 = \min\{x_1, x_2\}. \tag{6}$$

$$\text{Algebraic product: } x_1 \cdot x_2 = x_1, x_2. \tag{7}$$

$$\text{Bounded product: } x_1 \odot x_2 = 0 \wedge (x_1 + x_2 - 1). \tag{8}$$

$$\text{Drastic product: } x_1 \wedge x_2 = \begin{cases} x_1 & (x_2 = 1), \\ x_2 & (x_1 = 1), \\ 0 & (x_1, x_2 < 1). \end{cases} \tag{9}$$

2) t-conorm:

$$\text{Logical sum: } x_1 \vee x_2 = \max\{x_1, x_2\}. \tag{10}$$

$$\text{Algebraic sum: } x_1 \dot{+} x_2 = 1 \vee (x_1 + x_2). \tag{11}$$

$$\text{Bounded sum: } x_1 \oplus x_2 = 1 \vee (x_1 + x_2). \tag{12}$$

$$\text{Drastic sum: } x_1 \vee x_2 = \begin{cases} x_1 & (x_2 = 0), \\ x_2 & (x_1 = 0), \\ 1 & (x_1, x_2 > 0). \end{cases} \tag{13}$$

Various t-norm and t-conorm operators with parameters have been suggested by Schweizer, Yager, Dombi, Dubois, and Mizumoto (e.g., see [13]). Schweizer suggested t-norm T and t-conorm S as follows:

$$T = 1 - ((1 - x_1)^p + (1 - x_2)^p - (1 - x_1)^p (1 - x_2)^p (1 - x_2)^p)^{1/p}, \quad p > 0, \tag{14}$$

$$S = (x_1^p + x_2^p - x_1^p x_2^p)^{1/p}, \quad p > 0, \tag{15}$$

where p is a parameter which when changed can express various operators. For instance, for t-norm T, if $p = \infty$ it represents logical product $\wedge$; if $p = 1$ it represents algebraic product $\cdot$; if $p \to 0$ it represents drastic product $\curlywedge$. Similarly, t-conorm S can also express various operators.

Mean operators include the arithmetic mean, geometric mean, and the dual geometric mean:

$$\text{Arithmetic mean: } AM = \frac{x_1 + x_2}{2}. \tag{16}$$

$$\text{Geometric mean: } GM = \sqrt{x_1, x_2}. \tag{17}$$

$$\text{Dual geometric mean:} DGM = 1 - \sqrt{(1 - x_1)(1 - x_2)}. \tag{18}$$

The relationship among these fuzzy connectives is expressed as follows:

$$\curlywedge \leqq \odot \leqq \cdot \leqq \wedge \leqq GM \leqq AM \leqq DGM \leqq \vee \leqq \dot{+} \leqq \oplus \leqq \dot{\vee}. \tag{19}$$

In another respect, there are many studies of operators beyond t-norm and t-conorm operators. Zimmermann and Zysno [14] proposed multi-input and single-output operators which can express any operators between algebraic product "·" and algebraic sum "+," including mean operators. Zimmermann operators perform as follows:

$$y = \left[\prod_{j=1}^{n} (x_j)^{\sigma_j}\right]^{1-\gamma} \left[1 - \prod_{j=1}^{n} (1 - x_j)^{\sigma_j}\right]^{\gamma}, \tag{20}$$

$$\sum_{j=1}^{n} \sigma_j = n, \quad 0 \leqq \gamma \leqq 1, \tag{21}$$

where x_j, $j = 1, 2, \ldots, n$, are inputs, y is the calculation result; n ($n \geqq 1$) is the number of inputs, σ_j and γ are operator parameters (σ_j represents the weight among each input; γ represents the weight between the algebraic product and algebraic sum, e.g., if γ is small, the weight of the algebraic product is increased; if γ is large, the weight of the algebraic sum is increased).

Maeda and Murakami [10] extended the Zimmermann operators and proposed the following two operators:

$$\begin{gathered} y = \left[\prod_{j=1}^{n} (x_j)^{\sigma_j}\right]^{1-\gamma(x)} \left[1 - \prod_{j=1}^{n} (1 - x_j)^{\sigma_j}\right]^{\gamma(x)}, \\ \sum_{j=1}^{n} \sigma_j = n, \ \gamma(x) = a_0 + \sum_{j=1}^{n} a_j x_j, \ 0 \leqq \gamma \leqq 1, \end{gathered} \tag{22}$$

$$\begin{gathered} y = \left[\prod_{j=1}^{n} (x_j)^{\sigma_j(x)}\right]^{1-\gamma} \left[1 - \prod_{j=1}^{n} (1 - x_j)^{\sigma_j(x)}\right]^{\gamma}, \\ \sum_{j=1}^{n} \sigma_j(x) = n, \ \sigma_j(x) = b_0 + \sum_{k=1}^{n} b_{jk} x_k, \ 0 \leqq \gamma \leqq 1, \end{gathered} \tag{23}$$

where a_0, a_j, σ_j, b_{j0}, b_{ji}, and γ are operator parameters.

The convex formula combined with input variables $x_1, x_2, \ldots, x_n$ is represented by fuzzy connectives in (22) by extending γ in (21), so that by changing the input value, the fuzzy connectives in (22) can change the weight distribution between the algebraic product and algebraic sum. The operators are expressed in such a way that, when the input value is small, the algebraic product is increasingly important; when the input value is large, the algebraic sum is increasingly important.

On the other hand, the convex formula combined with input variables $x_1, x_2, \ldots, x_n$ is represented by the fuzzy connectives in (23) by expanding σ_j in formula (21), so that by changing the input value, the fuzzy connectives in (23) can change the weight distribution among $x_1, x_2, \ldots, x_n$. The operators are expressed in such a way that when the input value is small x_1 is increased; when the input value is large x_2 is increased.

Maeda proposed a learning function which, when input and output data are given, the most suitable parameter is sought by using a modified Newtonian method to minimize the sum of square error between the operator result and the output data. However, since we use a modified Newtonian method, only a local solution can be obtained during parametric search.

We now show the retrieval procedure of fuzzy retrieval using these fuzzy connectives. Consider retrieval queries P and Q:

$$P = p_1 \text{ and } p_2, \tag{24}$$

$$Q = q_1 \text{ or } q_2, \tag{25}$$

where p_1, p_2, q_1, and q_2 are fuzzy propositions expressed by the following fuzzy sets:

"employee = high salary and/or age about 30",

and can be regarded as retrieval queries.

Retrieval includes calculation of the correspondence of each data tuple stored in the database which satisfies the fuzzy proposition [8]. The t-norm and t-conorm ("and/or" operators, respectively) are employed to calculate the correspondence of retrieval queries, and the result is output in order high to low correspondence. First, the correspondence of W tuple $\boldsymbol{x}_i = (x_1, x_2)$, $i = 1, 2, \ldots, W$, stored in database that satisfy retrieval query formulas (24) and (25) is calculated as follows:

$$\mu_P(\boldsymbol{x}_i) = \mu_{p_1}(x_{i1}) \circledT \mu_{p_2}(x_{i2}), \tag{26}$$

$$\mu_Q(\boldsymbol{x}_i) = \mu_{q_1}(x_{i1}) \circledperp \mu_{q_2}(x_{i2}), \tag{27}$$

where $\mu_{p_1}(x_{i1})$, $\mu_{p_2}(x_{i2})$, and $\mu_{q_1}(x_{i1})$, $\mu_{q_2}(x_{i2})$, respectively, represent the membership functions of fuzzy propositions p_1, p_2 and q_1, q_2 of tuple $\boldsymbol{x}_i$; Ⓣ and ⊥⃝ represent the values of the t-norm and t-conorm, respectively. Once Ⓣ is set as one t-norm and ⊥⃝ is set as a t-conorm, we calculate $\mu_P(\boldsymbol{x}_i)$ and $\mu_Q(\boldsymbol{x}_i)$ of W tuple $\boldsymbol{x}_i = (x_{i1}, x_{i2})$. The result is obtained in order from high to low correspondence.

However, "and/or" in retrieval queries are defined as one t-norm and one t-conorm, respectively, so one cannot be sure of obtaining an ideal retrieval result for many various users. In addition, even though adjusted for each user by Zimmermann and Maeda fuzzy connectives, operators cannot express the drastic product and drastic sum. So we cannot say that "and/or" expressed by various users are completely expressed by the operators.

3. FUZZY CONNECTIVES WITH A LEARNING FUNCTION

3.1. Construction of Fuzzy Connectives with a Learning Function

We now propose new fuzzy connectives [16,17] to work as retrieval operators. These fuzzy connectives can express any operators between the drastic product and drastic sum. When the user estimate

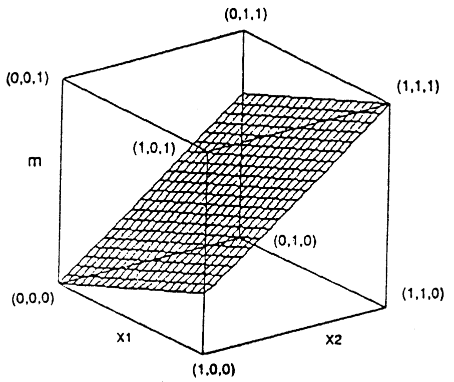


Fig. 1. An example of m in a fuzzy connective with a learning function.

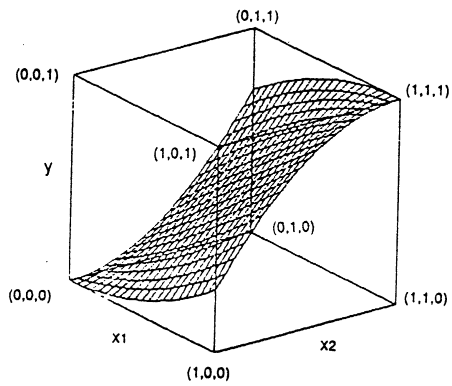


Fig. 2. An example of a fuzzy connective with a learning function.

of retrieval results is given, fuzzy connective parameters are adjusted to minimize the sum of square error between the user estimate and the fuzzy connective output. These fuzzy connectives are termed "fuzzy connectives with a learning function" (FCLFs).

Fuzzy connectives with a learning function are expressed as follows:

$$f(x) = m \cdot S + (1 - m) \cdot T, \tag{28}$$

$$\begin{aligned} m &= p_1 - \sum_{j=1}^{n} (p_1 - p_{j+1}) x_j \\ 0 &\leqq p_1, \ldots, p_{n+1} \leqq 1, \ 0 \leqq -(n-1)p_1 + \sum_{j=1}^{n} p_j \leqq 1, \end{aligned} \tag{29}$$

where T and S represent t-norms and t-conorms expanded to be multi-input. When a Schweizer t-norm and t-conorm are used, T and S are represented respectively as follows:

$$T = 1 - \left[1 - \prod_{j=1}^{n} \{1 - (1. - x_j)^{p_{n+2}}\} \right]^{1/p_{n+2}}, \quad p_{n+2} > 0, \tag{30}$$

$$S = \left[1 - \prod_{j=1}^{n} (1 - x_j^{p_{n+3}}) \right]^{1/p_{n+3}}, p_{n+3} > 0, \tag{31}$$

where $p_1, p_2, \ldots, p_{n+3}$ are parameters.

The FCLFs of (28) can be combined linearly with a T and S by using the value of m, which can be calculated using (29) from input variables $x_1, x_2, \ldots, x_n$. Thus, according to the input value, FCLFs can change the weight between T and S. Figure 1 shows an example of m with two inputs (x_1 and x_2), where, when x_1 and x_2 are small, m is small; as x_1 and x_2 increase, the parameters are set to make m larger. Because input x_2 is more important than input x_1, the gradient of m in plane $x_1 = 0$ is bigger. An example of the FCLF input/output relationship using m from Fig. 1 is depicted in Fig. 2. According to (29), when inputs x_1 and x_2 are small, FCLFs become a t-norm weight operator; when inputs x_1 and x_2 are large, FCLFs become a t-conorm weight operator. Because

Table 1. Relationship of Operator and Parameters in Fuzzy Connectives with a Learning Function

Parameter					Fuzzy connective
p_1	p_2	p_3	p_4	p_5	
0	0	0	any	$\to 0$	Drastic product
1	1	1	$\to 0$	any	Drastic sum
0.5	0.5	0.5	∞	∞	Arithmetic mean operator

input x_2 has more weight than input x_1, the t-conorm is more important for the FCLF when input x_2 is large.

FCLFs can be expressed as parametric t-norms and t-conorms. Compared to Zimmermann and Maeda fuzzy connectives, which can only represent operators between the algebraic product and algebraic sum, FCLFs can represent any operators between the drastic product and drastic sum. When parameters $p_1, p_2, \ldots, p_5$ with two inputs (x_1 and x_2) are changed, the FCLF can express various operators (see Table 1). Various t-norm and t-conorm operators can be expressed by changing parameters p_4 and p_5. In addition, when convex combination (28) is used, as with changing t-norms and t-conorms, FCLFs can represent linear mean operators.

Just as with the conventional fuzzy connectives used to calculate (26) and (21), when FCLFs are used, the correspondence of a retrieval query can be calculated as follows:

$$\mu_P(\boldsymbol{x}_i) = \mu_{p_1}(x_{i1}) \otimes \mu_{p_2}(x_{i2}), \tag{32}$$

$$\mu_Q(\boldsymbol{x}_i) = \mu_{q_1}(x_{i1}) \otimes \mu_{q_2}(x_{i2}), \tag{33}$$

where $\mu_{p_1}(x_{i1})$, $\mu_{p_2}(x_{i2})$, and $\mu_{q_2}(x_{i1})$, $\mu_{q_2}(x_{i2})$, respectively, represent the membership function of fuzzy propositions p_1, p_2 and q_1, q_2 related to tuple $\boldsymbol{x}_i = (x_{i1}, x_{i2})$, $i = 1, 2, \ldots, W$; $\otimes$ represents an FCLF, which here can express the t-norm and t-conorm included by operators between the drastic product and drastic sum. Thus, the t-norm in (32) and the t-conorm in (33) can be represented by the same sign: $\otimes$.

3.2. Method for Adjusting Parameters of the FCLFs

When data $(\boldsymbol{x}_i, y_i) = (x_{i1}, \ldots, x_{in}, y_i)$, $i = 1, 2, \ldots, N$, are given, the FCLF parameters are adjusted to minimize the sum of square error. We now describe this adjustment method.

Sum of square error E between operator result $\hat{y}$ and output y is defined as follows:

$$E = \frac{(\hat{y} - y)^2}{2}. \tag{34}$$

We next calculate the correction value for parameters p_j, $j = 1, 2, \ldots, n+3$, which makes E smallest. To minimize E, the effect of a minute change ($\partial E/\partial p_j$) in parameter p_j on difference E is expressed by the effect of a minute change ($\partial \hat{y}/\partial p_j$) in p_j on result $\hat{y}$ and the effect ($\partial E/\partial \hat{y}$) of result $\hat{y}$ on E as follows:

$$\frac{\partial E}{\partial p_j} = \frac{\partial E}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial p_j}, \tag{35}$$

Name of hotel	Accommodation (¥)	Time to station (min)
A	15000	20
B	15000	15
C	15000	10
D	15000	3
E	12000	20
F	12000	15
G	12000	10
H	12000	3
I	10000	20
J	10000	15
K	10000	10
L	10000	3
M	5000	20
N	5000	15
O	5000	10
P	5000	3

Fig. 3. Hotel database.

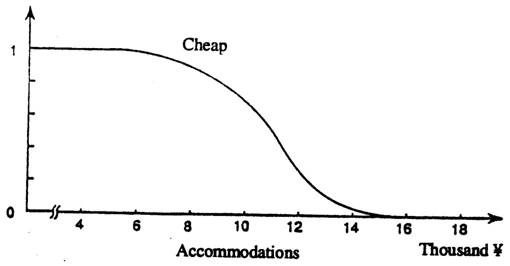


Fig. 4. Membership function of "cheap."

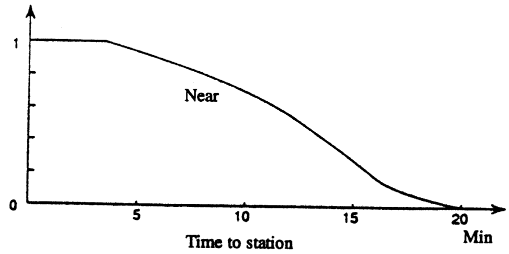


Fig. 5. Membership function of "near."

which can be obtained from (34) as

$$\frac{\partial E}{\partial \hat{y}} = \hat{y} - y. \tag{36}$$

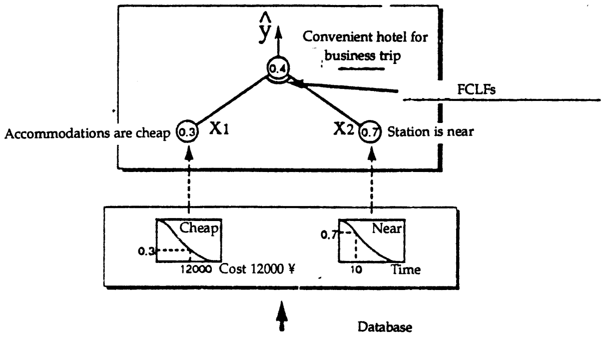
According to (28)–(31), the right-hand side ($\partial \hat{y}/\partial p_j$) of (35) can be obtained from (28)–(31) as follows:

$$\frac{\partial \hat{y}}{\partial p_1} = \left[1 - \sum_{j=1}^{n} x_j\right] \cdot (S - T), \tag{37}$$

$$\frac{\partial \hat{y}}{\partial p_1} = x_{j-1} \cdot (S - T), \; j = 2, 3, \ldots, n+1, \tag{38}$$

$$\frac{\partial \hat{y}}{\partial p_{n+2}} = (1 - m) \cdot \frac{\partial T}{\partial p_{n+2}}, \tag{39}$$

$$\frac{\partial \hat{y}}{\partial p_{n+3}} = m \cdot \frac{\partial S}{\partial p_{n+3}}, \tag{40}$$



Hotel name	Accommodation	Time
A	15000	20
B	15000	15
C	15000	10
:	:	:
P	5000	3

Fig. 6. Conceptual scheme of the retrieval experiment.

where T and S are the Schweizer t-norm and t-conorm, respectively, so that $\partial T/\partial p_{n+2}$ and $\partial S/\partial p_{n+3}$ in (39) and (40) can be obtained as follows:

$$\begin{aligned}\frac{\partial T}{\partial p_{n+2}} =&(1-T)\Big(\frac{1}{p_{n+2}^2}\log(f) + (1-x_2)^{p_{n+2}}\\&\cdot\log(1-x_2) - \frac{1}{p_{n+2}\cdot f}\cdot((1-x_1)^{p_{n+2}}\\&\log(1-x_1) - (1-x_1)^{p_{n+2}}(1-x_2)^{p_{n+2}}\\&\cdot\log(1-x_1)(1-x_2))\Big),\end{aligned} \tag{41}$$

$$\begin{aligned}\frac{\partial S}{\partial p_{n+3}} =&S\Big(\frac{1}{p_{n+3}^2}\log(g) + \frac{1}{p_{n+3}\cdot g}\\&\cdot(x_1 p_{n+3}\log(x_1) + p_{n+3}^2\log(x_2)\\&- x_1 p_{n+3} x_2 p_{n+3}\log(x_1, x_2))\Big),\end{aligned} \tag{42}$$

$$\begin{aligned}f =&(1-x_1)^{p_{n+2}} + (1-x_2)^{p_{n+2}}\\&-(1-x_1)^{p_{n+2}}(1-x_2)^{p_{n+2}},\end{aligned} \tag{43}$$

$$g = x_1 p_{n+3} + x_2 p_{n+3} - x_1 p_{n+3} x_2 p_{n+3}. \tag{44}$$

$\partial E/\partial p_j$ can be calculated by using (36)–(44). Thus E can be minimized by using the steepest descent method [15] and changing parameter p_j in the opposite direction in $\partial E/\partial p_j$. E can also be minimized by using constant α (> 0) and changing parameter $\partial\hat{y}/\partial p_j$ repeatedly in the direction of the following formula:

$$\Delta p_j = -\alpha\cdot\frac{\partial E}{\partial p_j}. \tag{45}$$

Table 2. Estimate Given by User

	Hotel	Extent that accommodation is cheap	Extent that station is near	Estimate a	Estimate b	Estimate c	Estimate d
Learning/using data							
	C	0.0	0.7	0.0	0.7	0.3	0.3
	E	0.3	0.0	0.0	0.3	0.1	0.0
	F	0.3	0.3	0.0	0.8	0.3	0.2
	G	0.3	0.7	0.1	0.9	0.5	0.5
	J	0.7	0.3	0.1	0.9	0.5	0.5
	K	0.7	0.7	0.2	1.0	0.7	0.8
	L	0.7	1.0	0.7	1.0	0.8	1.0
	N	1.0	0.3	0.3	1.0	0.7	0.8
Sum of square error of initial parameter				0.52091	0.52091	0.02361	0.00481
Checking/using data							
	A	0.0	0.0	0.0	0.0	0.0	0.0
	B	0.0	0.3	0.0	0.3	0.1	0.0
	D	0.0	1.0	0.2	0.9	0.5	0.6
	H	0.3	1.0	0.3	1.0	0.6	0.8
	I	0.7	0.0	0.0	0.7	0.4	0.3
	M	1.0	0.0	0.2	0.9	0.5	0.5
	O	1.0	0.7	0.7	1.0	0.9	1.0
	P	1.0	1.0	1.0	1.0	1.0	1.0
Sum of square error of initial parameter				0.25705	0.32705	0.02705	0.00955

Table 3. Parameters After Learning

Parameter	User A	User B	User C	User D
p_1	0.0044	0.9791	0.4255	0.0638
p_2	0.0078	0.9882	0.5417	0.5168
p_3	0.0103	0.9939	0.4087	0.5470
p_4	1.4362	0.1943	0.7912	1.2241
p_5	0.1973	0.5665	1.0103	0.8147

4. RETRIEVAL EXPERIMENT USING FCLFs

We now use FCLFs to do a retrieval experiment to demonstrate their usefulness. We chose a simple trial system:

Retrieve: convenient hotel for business trip.

from a hotel information database. Data from 16 hotels on cost of accommodations (in yen) and the time to a transportation station (in minutes) were stored in the database (Fig. 3). We used the following retrieval query:

Convenient hotel for business trip = "accommodations are cheap" and/or "the station is near,"

Table 4. Estimated and Retrieval Results

Order	User A				User B			
	Estimate		Retrieval result		Estimate		Retrieval result	
1	P	1.00	P	1.00	P	1.00	P	1.00
2	O	0.70	O	0.70	O	1.00	O	1.00
3	O	0.70	O	0.70	O	1.00	O	1.00
4	N	0.30	H	0.31	D	1.00	H	1.00
5	N	0.30	H	0.31	D	1.00	H	1.00
6	M	0.20	K	0.21	H	1.00	D	0.99
7	K	0.20	G	0.08	M	0.90	M	0.99
8	D	0.20	J	0.08	J	0.90	K	0.97
9	I	0.10	E	0.03	G	0.90	G	0.92
10	G	0.10	D	0.01	D	0.90	I	0.92
11	1	0.00	M	0.01	E	0.80	E	0.79
12	E	0.00	C	0.01	I	0.70	C	0.69
13	E	0.00	I	0.00	C	0.70	I	0.69
14	C	0.00	E	0.00	E	0.30	B	0.30
15	B	0.00	B	0.00	B	0.30	E	0.29
16	A	0.00	A	0.00	A	0.00	A	0.00

A square around a letter represents adjusting/using data.

Table 5. Sum of Square Error Between Estimated and Retrieval Results

	User A	User B	User C	User D
Adjusting/using data				
Fuzzy connective with learning function	0.0010	0.0010	0.0025	0.0040
Maeda fuzzy connective	–	–	0.0517	0.0453
Verifying/using data				
Fuzzy connective with learning function	0.03652	0.0084	0.0073	0.0052
Maeda fuzzy connective	–	–	0.3379	0.3510

where "cheap" and "near" are membership functions, depicted in Figs. 4 and 5.

In this retrieval system, we adopt an FCLF to express "and/or," adjust the FCLF parameters by the user estimate of "convenient hotel for business trip," and retrieve the satisfied result.

Figure 6 depicts the conceptual scheme of this retrieval experiment. There were five retrieval steps:

1. Users determine the form of membership functions "cheap" and "near."
2. Users give a 16-hotel estimate of "convenient hotel for business trip," eight of which are learning/using parameter; the others are checking/using data used to check the retrieval result.
3. Using the eight learning/using data, we adjust the FCLF parameters by the steepest descent method.
4. Using "and/or" after learning, we calculate the correspondence of the 16-hotel retrieval queries from (32) and (33) and output the hotels in order of high to low correspondence.
5. We check the retrieval result using the checking/using data.

To demonstrate the usefulness of FCLFs, four users supply estimates from the following four points of view: a) with the idea of a drastic product; b) with the idea of a drastic sum; c) with the idea of an arithmetic mean operator; d) with the view that, when each factor is small, we emphasize "and," and when each factor is large, we emphasize "or."

The four user estimates for learning and checking data for each hotel are shown in Table 2. The initial value of the parameters in (29)–(31) are set at $p_1 = 0.0$, $p_2 = p_3 = 0.5$, $p_4 = 1.0$, $p_5 = 1.0$. The parameters of FCLFs are adjusted with constant $\alpha = 0.01$ in (45) and the learning/using data in Table 2. The parameters after learning are given in Table 3. The estimated and retrieval result are given in Table 4 in order of value for two users (A and B), from which we can conclude that, not only for learning/using data, but also for checking/using data, the retrieval results agree quite well with the estimates, especially for drastic product users and drastic sum users, and the expected results can be retrieved.

We then conducted the same retrieval experiment by replacing the fuzzy connectives with Maeda fuzzy connectives. We compare these results with that of a retrieval experiment using FCLFs. Maeda and Murakami [10] described development of the fuzzy connectives in (22) in more detail than that in (23), so we just compare the fuzzy connectives in (22) to an FCLF. To make the experimental conditions the same as those for FCLFs, the initial parameters in (22) were set at $a_0 = 0.0$, $a_1 = a_2 = 0.5$, $\sigma_1 = \sigma_2 = 1.0$, and the learning coefficient in the modified Newtonian method was set at 0.01. The sum of square error between the estimated and retrieval results are given in Table 5. Sum of square of error TE is calculated as follows:

$$TE = \frac{\sum_{i=1}^{8} (\check{y} - y_i)^2}{2}, \tag{46}$$

where y_j is the estimate of learning/using data or checking/using data, and $\check{y}_i$ is the retrieval result using Maeda fuzzy connectives or FCLFS.

Table 5 compares the sum of square errors using FCLFs to those in Table 2, and shows that it is small not only for learning/using data but also for checking/using data. This demonstrates the usefulness of FCLFs. In terms of Maeda fuzzy connectives users A and B's estimates are based on the drastic product and drastic sum, respectively, so that Maeda fuzzy connectives cannot express this estimate. Comparing users C and D, the sum of square error using FCLFs is smaller than that when using Maeda fuzzy connectives. We can thus conclude that FCLFs express user "and/or" operators better.

5. CONCLUDING REMARKS

We have described fuzzy connectives with a learning function which learn fuzzy connective parameters to express user retrieval requests by the steepest descent method and have demonstrated their usefulness in an experiment. The experiment showed that FCLFs can represent any operators between the drastic product and drastic sum; they can also represent various user "and/or" operators. It is now important to compare this method with other retrieval methods [18] and to discuss FCLFs in multi-layers [19] to express more complicated retrieval queries.

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