

Three-dimensional Motion Analysis for Gesture Recognition Using Singular Value Decomposition

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Abstract – A gesture is a form of non-verbal communication in which visible bodily actions communicate particular messages, either in place of speech or together and in parallel with spoken words. Gestures are important in the communication between human and human. It will make a robot more human-friendly to enable it to communicate with human by gestures. Our research addresses to develop a method to recognize human gestures for a guide robot that can be used in hospitals, welfare facilities, and etc. In this paper, firstly, a novel 3D motion analysis algorithm for gesture recognition using singular value decomposition (SVD) is proposed. An experiment, in which five gestures is included, is carried out to testify the effectiveness of the algorithm. The experiment results indicate that the proposed algorithm is applicable for the guide robot to recognize human gestures in guidance.

Index Terms – Motion Analysis, Gesture Recognition, Singular Value Decomposition (SVD), Guide Robot.

I. INTRODUCTION

Recently, human-friendly robots are expected to support human daily life at home, office, medical treatment and welfare scene with rapid development of declining children population and increasing aging community. Unlike industrial robots working in factories, human-friendly robots should be able communicate with human in the ways that human communicate with each other. Human communicate in nonverbal ways, such as gestures, facial expressions, posture, appearance, listening and eye contact, as well as in verbal ways, such as speaking and writing [1]. In our study, gestures are taken into consideration because viewing gestures during face-to-face communication affects speech perception and comprehension. Gestures can alter the interpretation of speech, disambiguate speech, increase comprehension and memory, and convey information not delivered by speech [2]. A robot will be much more intimate if it can understand hand gestures in its communication with a human.

In previous studies, authors and colleagues have developed a guide robot (Fig.1) that can be used in hospitals, welfare facilities, and etc. So far, the trajectory planning method for

guidance has been developed for the guide robots based on the fuzzy reasoning and voice communication ability has been equipped with it, which makes it be able to understand a human’s instructions in guidance [3]. Furthermore, In order to guide persons with visual impairment, guidance leads, which conduct bidirectional force information between user and the robot, have been developed and their usability and safety have been evaluated by psychological experiment [4]. In this paper, a novel gesture recognition method is proposed to enable the guide robot to communicate with humans with gestures in guidance.



Fig. 1 Guide Robot

Gesture recognition has been studied extensively and there have been varied approaches to handle gesture recognition, ranging from mathematical models based on hidden Markov chains to tools or approaches based on soft computing [5]. In this paper, a novel gesture recognition method using singular value decomposition (SVD) is proposed to identify 3-dimensional gesture motions and the proposed method is testified in a hand gesture recognition experiment.

II. GESTURE RECOGNITION BASED ON MOTION ANALYSIS USING SINGULAR VALUE DECOMPOSITION

A. Singular Value Decomposition

Suppose M is an m -by- n matrix. Then there exists a factorization of the form

$$M = U \Sigma V^T \tag{1}$$

where $U = [u_1, u_2, \dots, u_m]$ is an m -by- m unitary matrix, the matrix Σ is m -by- n diagonal matrix with nonnegative real

numbers on the diagonal, and V^T denotes the conjugate transpose of $V = [v_1, v_2, \dots, v_n]$, an n-by-n unitary matrix. Such a factorization is called singular value decomposition (SVD) of M . The diagonal entries of Σ are known as the singular values of M . The matrix U contains the left singular vectors of M and the matrix V contains the right singular vectors of M .

The SVD is an important factorization of a rectangular real or complex matrix, with many applications in signal processing and statistics. Applications which employ the SVD include computing the pseudoinverse, least squares fitting of data, matrix approximation, and determining the rank, range and null space of a matrix [6-7]. Recently, the SVD have been utilized in time-series data analysis for knowledge discovery [8] and motion analysis to extract similarities and differences in human behavior [9].

B. Motion Analysis Using SVD

Suppose that a time series $X = \{x_1, x_2, \dots, x_m\}^T$ contains the x coordinate values of a point on the hand while a hand gesture G is performed. $X_j (j=1,2,\dots,n)$ denote the x coordinate values of the j th performance of the gesture. Then matrix M_X is defined as a collective of the change of x coordinate values of the gesture.

$$M_X = [X_1, X_2, \dots, X_n] \quad (2)$$

The matrix M_X can be decomposed into a product of U , Σ and V . Intuitively, the left singular vectors in U form a set of patterns of M and the diagonal values in matrix Σ are the singular values, which can be thought of as scalar by which each corresponding left singular vectors affect matrix M_X .

Let us denote the singular values and the left singular vectors as $\{(\delta_1, u_1), (\delta_2, u_2), \dots, (\delta_l, u_l)\}$, in descending order of the singular values. The parameter l represents the number of representative patterns under consideration. The greater the singular value is, the more dominant the corresponding pattern is. If a singular value is small, then the corresponding pattern can be considered to be a noise component. Therefore, the left singular vectors u_1, u_2, \dots, u_l of M_X represent the change patterns of the x coordinate values on this point of the hand gesture.

C. Gesture Recognition Based on the Similarity between Gestures

We proposed two kinds of motion analysis methods for gesture recognition using SVD. Specially, the second method we proposed will be useful for developing guide robot in near future. Therefore, we only give a brief introduction to the first method and explain the second method in more detail.

1) Method for Similarity between Gesture Distances

The first method is to recognize gestures based on the similarity between gesture distances. Suppose that there are k measurement points $\{P_1, P_2, \dots, P_k\}$. On point P_i , the measured data series of gesture R is denoted as τ_i^R . Suppose that the measured data series are divided into $\tau_{TRD}^{i,R}$ as reference data series and $\tau_{CHD}^{i,R}$ as data series to be recognized. In the first

method, the gesture pattern is extracted according to the method in Ref. [8]. Let us denote the left singular vectors of $\tau_{TRD}^{i,R}$ and $\tau_{CHD}^{i,R}$ are $u_{j,TRD}^{i,R} = (u_{1,TRD}^{i,R}, u_{2,TRD}^{i,R}, \dots, u_{q,TRD}^{i,R})$ and $u_{j,CHD}^{i,R} = (u_{1,CHD}^{i,R}, u_{2,CHD}^{i,R}, \dots, u_{q,CHD}^{i,R})$. Three kinds of similarity between gesture data series are defined in Eq.(3), Eq.(4) and Eq.(5).

$$r_i(u_{TRD}^{i,R}, u_{CHD}^{i,R}) = \frac{1}{3lq} \sum_{k=1}^3 \sum_{j=1}^l \sum_{h=1}^q |u_{hj,TRD}^{i,R} - u_{hj,CHD}^{i,R}| \quad (3)$$

$$r_i(u_{TRD}^{i,R}, u_{CHD}^{i,R}) = \frac{1}{3lq} \sum_{k=1}^3 \sum_{j=1}^l \sqrt{\sum_{h=1}^q (u_{hj,TRD}^{i,R} - u_{hj,CHD}^{i,R})^2} \quad (4)$$

$$r_i(u_{TRD}^{i,R}, u_{CHD}^{i,R}) = \frac{1}{3lq} \sum_{k=1}^3 \sum_{j=1}^l \left| \sum_{h=1}^q u_{hj,TRD}^{i,R} - \sum_{h=1}^q u_{hj,CHD}^{i,R} \right| \quad (5)$$

Based on the similarity between gestures, gestures is identified in two ways shown in Eq.(6) and Eq.(7)

$$R^* = \left\{ R_i \mid \max_i \left\{ \text{count} R_i \mid \min_j r_i(u_{TRD}^{i,R}, u_{CHD}^{i,R}) \right\} \right\} \quad (6)$$

$$R^* = \left\{ R_i \mid \min_j \sum_{i=1}^k r_i(u_{TRD}^{i,R}, u_{CHD}^{i,R}) \right\} \quad (7)$$

We applied the method for similarity between gesture coordinates to simple gesture examples. As a result, we obtained high recognition more than 90% with the combination of the similarity defined in Eq.(3) and the identification method in Eq.(6).

2) Method for Similarity between Gesture Vectors

As for the second method, gesture recognition is based on the similarity between gesture vectors. Consider the data series in M_X of Eq.(2). If one of the data series X_p is replaced by another data series X^* , as shown in Eq.(8), the singular values and left singular vectors of M_X^* will be different from those of M_X .

$$M_X^* = [X_1, X_2, \dots, X_{p-1}, X^*, \dots, X_n] \quad (8)$$

The difference between the left singular vectors of M_X and M_X^* is determined by how different X^* is from the other data series because the left singular vectors represent the patterns of M_X and M_X^* . The patterns of M_X will change more when X_p is replaced by a quite dissimilar data series than when it is replaced by a similar one. Therefore, the difference between the left singular vectors of M_X and M_X^* can be considered as a measure of the difference between X^* and the other data series. Furthermore, if X^* comes from another kind of hand gesture, the difference can be utilized as a criterion for judging whether X^* comes from the same kind of hand gesture as the other data series.

If the singular values and the left singular vectors are denoted as $\{(\delta_1, u_1), (\delta_2, u_2), \dots, (\delta_l, u_l)\}$, in descending order

of the singular values. The difference between matrix M_X and X^* is defined as

$$D(M_X, X^*) = \frac{1}{l} \sum_{j=1}^l d(u_j, u_j^*) \quad (9)$$

where $d(u_j, u_j^*)$ is the difference between u_j and u_j^* .

Suppose $u_j = \{u_{j1}, u_{j2}, \dots, u_{jq}\}$, then $d(u_j, u_j^*)$ is defined in three methods in this paper as shown in Eq.(10), Eq.(11), and Eq.(12). The effectiveness of the three methods will be verified in the next section.

$$d(u_j, u_j^*) = \frac{1}{q} \left| \sum_{h=1}^q u_{jh} - \sum_{h=1}^q u_{jh}^* \right| \quad (10)$$

$$d(u_j, u_j^*) = \frac{1}{q} \sum_{h=1}^q |u_{jh} - u_{jh}^*| \quad (11)$$

$$d(u_j, u_j^*) = \frac{1}{q} \sum_{h=1}^q \sqrt{(u_{jh} - u_{jh}^*)^2} \quad (12)$$

So far, on one point's x coordinate values, the difference between an unknown data series X^* and a gesture's patterns is defined. If there are more than one points, P_1, P_2, \dots, P_w and on each point 3-dimensional values are measured, the difference on one point is defined as the average difference of the 3 dimensions in Eq.(13). And the difference between a gesture G and an unknown gesture G^* is defined as the average difference of the differences on all the measurement points in Eq.(14).

$$D_{P_i}(G, G^*) = \frac{D_{P_i}(M_X, X^*) + D_{P_i}(M_Y, Y^*) + D_{P_i}(M_Z, Z^*)}{3} \quad (13)$$

$$D(G, G^*) = \frac{1}{w} \sum_{i=1}^w D_{P_i}(G, G^*) \quad (14)$$

where D_{P_i} is the difference of one dimension on point P_i calculated by Eq.(9)

Suppose that G^* is one of a group of hand gestures $\{G_1, G_2, \dots, G_s\}$, then G^* can be identified as the gesture whose difference from G^* is minimum.

$$G^* = \{G_i \mid D(G_i, G^*) = \min_{1 \leq j \leq s} D(G_j, G^*)\} \quad (15)$$

A gesture recognition experiment was carried out to testify the second method.

III. GESTURE RECOGNITION EXPERIMENT

A. Three Dimensional Motion Measurement

In the experiment, five kinds of hand gestures, CH (Come here), GA (Go away), GR (Go right), GL (Go left), and CD (Calm down), were performed by two subjects, SW and ST. These gestures are considered to be useful in guidance. The gestures were performed in a 50cm×50cm×50cm cubic space, whose zero point and coordinate system are shown in Fig.2. The motions of the hand gestures are measured with Move-tr/3D and GE60/W (Library, Tokyo, Japan).

One gesture was executed 9 times by each subject. Data of the first 5 times execution were used as patterns of the gesture. Data of last 4 times were used to be distinguished. Five markers, M1 on the tip of the thumb, M2 on the tip of the middle finger, M3 on the tip of the little finger, M4 on the thumb-side of the wrist and M5 on the little finger side of the wrist, were measured.

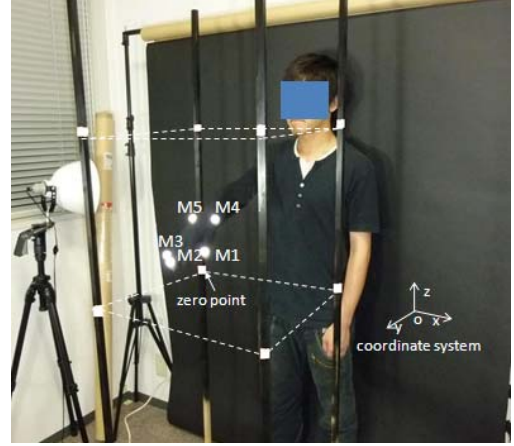


Fig. 2 Experiment scene

B. Data Preprocessing

In the proposed method, all of the time series must have the same number of data in order to compose matrix M as shown in Eq.(2). However, the lengths of the hand gestures in real measurement are different according to the kinds of gestures and the subjects to perform them. Therefore, preprocessing is necessary to make the data series have the same number of data. On the other hand, speed is an important factor of gesture. Gesture meanings could vary with its speed. Thus, each gesture should have a range of speed.

In this paper, the number of data was set to be the average number. If a data series contains more data than the average number, data are deleted from the data series at the same interval. If a data series contains fewer data than the average number, data are interpolated in to the data series at the same interval. The interpolated data are calculated using quadric interpolation.

IV. RESULTS AND DISCUSSIONS

The data of M2 when subject SW performed the five kinds of gestures are shown in Fig.3 as examples. It can be seen that the lengths of data series were different. In the experiment, the average number of data of SW's gestures is 100.3 (SD: 12.8) and that of ST's gestures is 150.3 (SD: 16.4). The average number of SW and ST's gestures is 125.2 (SD: 29.0).

In order to testify the effectiveness of the proposed method, gesture recognition was carried out in three gesture groups, recognizing SW's gestures among SW's gesture patterns, recognizing ST's gestures among ST's gesture patterns, and recognizing the two subjects' gestures among their gesture patterns. The numbers of data are set to be 100, 150 and 125, respectively. For each gesture's 9 times

execution, the first 5 times of execution was used to extract gesture patterns and the other 4 times execution was used to

be recognized. Therefore, parameter n in Eq.(2) was 5. Parameter p was also set to be 5, which means that the last column of M was replaced to form M^* . Parameter w , the number of measurement points, is 5. Parameter s , the number of patterns, in Eq.(15) is 5 in the within-subject recognition among SW or ST's gestures and 10 in the intersubject recognition among both SW and ST's recognition. These patterns were denoted as SW_CH, SW_GA, SW_GR, SW_GL, SW_CD for subject SW's patterns, and ST_CH, ST_GA, ST_GR, ST_GL, ST_CD for subject ST's patterns. Only the singular vector corresponding to the biggest singular value was considered. Thus, parameter k in Eq.(9) was 1.

A. Intersubject Recognition Results

Table I shows the recognition results among both of the two subjects' gesture patterns based on the three kinds of difference definitions of Eq.(10), Eq.(11) and Eq.(12). For example, 4 times execution of SW_GR were recognition as SW_GR twice, ST_CH once and ST_GA once, based on the difference definition of Eq.(10). The recognition results suggest that difference definitions of Eq.(11) and Eq.(12) led to relatively higher correct recognition rates while the correct rate of the recognition based on difference of Eq.(10) was very low. Therefore, difference definitions of Eq.(11) and Eq.(12) are more feasible in gesture recognition. The following recognition results shown in this paper are all based on Eq.(11)

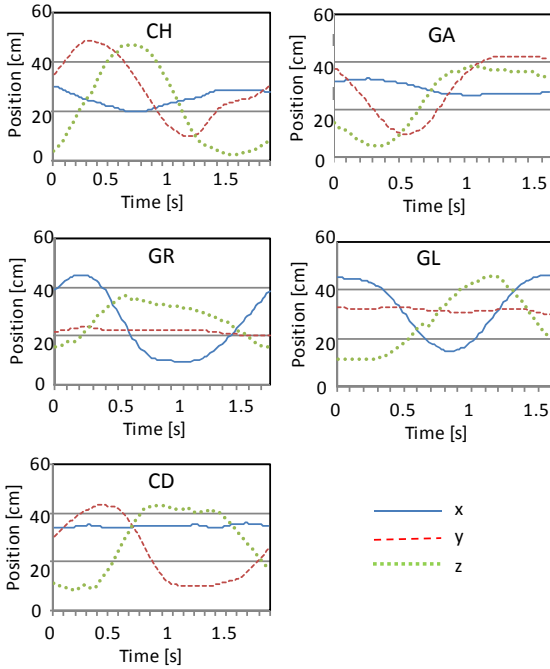


Fig. 3 Gesture examples (M2 of SW)

TABLE I
RECOGNITION RESULTS OF THREE DIFFERENCE DEFINITIONS AMONG SW AND ST'S GESTURES

Gesture Motions	Recognition Results		
	Difference of Eq.(10)	Difference of Eq.(11)	Difference of Eq.(12)
SW_CH	SW_CH: 1 ST_CH: 1 ST_CD: 2	SW_CH: 2 SW_GA: 2	SW_CH: 1 SW_GA: 2 ST_GA: 1
SW_GA	SW_GA: 3 ST_GR: 1	SW_GA: 4	SW_GA: 4
SW_GR	SW_GR: 2 ST_CH: 1 ST_GA: 1	SW_GR: 3 ST_CD: 1	SW_GR: 3 ST_CD: 1
SW_GL	SW_GL: 1 ST_CH: 1 ST_GA: 1	SW_GL: 3 SW_GR: 1	SW_GL: 3 SW_GR: 1
SW_CD	ST_CH: 2 ST_GA: 1 ST_GR: 1	SW_CD: 3 SW_GR: 1	SW_CD: 3 SW_GR: 1
ST_CH	ST_CH: 2 ST_CD: 2	ST_CH: 2 ST_CD: 2	ST_CH: 2 ST_CD: 2
ST_GA	SW_GA: 1 ST_CH: 2 ST_GL: 1	ST_GA: 4	ST_GA: 4
ST_GR	ST_GR: 3 SW_GA: 1	ST_GR: 4	ST_GR: 4
ST_GL	SW_GA: 1 SW_CH: 3	ST_GL: 3 SW_GR: 1	ST_GL: 2 SW_GR: 2
ST_CD	ST_CH: 4	ST_CD: 4	ST_CD: 4
Correct Rate	30%	80%	75%

Correct recognition is *italic*

B. Within-subject Recognition Results

The recognition results of within-subject recognition among SW or ST's gestures are shown in Table II. The correct recognition rate of SW's gestures is quite lower than that of ST's gestures. This is because that ST's motion in the experiment had a relatively high reproducibility. These results verified the effectiveness of the proposed gesture recognition method.

TABLE II
RECOGNITION RESULTS WITHIN SW'S AND ST'S GESTURES

Gesture Motions	Recognition Results	
	SW	ST
CH	<i>CH: 1</i> GA: 1 GR: 1 CD: 1	<i>CH: 2</i> CD: 2
GA	<i>GA: 4</i>	<i>GA: 4</i>
GR	<i>GR: 3</i> GL: 1	<i>GR: 4</i>
GL	<i>GL: 3</i> GR: 1	<i>GL: 4</i>
CD	<i>CD: 3</i> GR: 1	<i>CD: 4</i>
Correct rate	70%	90%

Correct recognition is *italic*

Similar to speech and handwriting, gestures vary between individuals, and even for the same individual between different instances. There are great individual differences in gesture comprehension and gesture production. As shown in Table I, the recognition results based on difference definition of Eq.(11) illustrated that gestures of one person were seldom recognized as those of the other person. Individual differences in gesture production include not only speed, but also direction, the range of movement and so on. Therefore, it is necessary to have sufficient data in the gesture patterns to improve the recognition accuracy.

As for the incorrect recognitions, although the motion was incorrectly recognized as a gesture different from the intended one, its motion data were quite similar to those in the gesture pattern as which it was incorrectly recognized. For example, the gestures GR and GL have completely different meanings, but their motions are very similar in that the hand waves left and right. Their difference lies in whether the hand moves faster from left to right, or from right to left. Sometimes even humans make mistakes in distinguishing them from each other.

TABLE III
RECOGNITION RESULTS BASED ON ONE MEASUREMENT POINT

Gesture Motions	Recognition Results				
	M1	M2	M3	M4	M5
SW_CH	<i>SW_CH: 1</i> <i>SW_GA: 1</i> <i>SW_CD: 1</i> <i>ST_GA: 1</i>	<i>SW_CH: 1</i> <i>SW_GA: 1</i> <i>SW_CD: 1</i> <i>ST_GA: 1</i>	<i>SW_CH: 1</i> <i>SW_GA: 1</i> <i>SW_CD: 1</i> <i>ST_GA: 1</i>	<i>SW_GA: 2</i> <i>SW_GR: 1</i> <i>ST_GA: 1</i>	<i>SW_CH: 1</i> <i>SW_GA: 2</i> <i>SW_GR: 1</i>
SW_GA	<i>SW_GA: 4</i>	<i>SW_GA: 4</i>	<i>SW_GA: 2</i> <i>SW_GL: 1</i> <i>ST_GL: 1</i>	<i>SW_GA: 4</i>	<i>SW_GA: 4</i>
SW_GR	<i>SW_GR: 3</i> <i>SW_GL: 1</i>	<i>SW_GR: 3</i> <i>SW_GL: 1</i>	<i>SW_GR: 3</i> <i>SW_GA: 1</i>	<i>SW_GR: 3</i> <i>SW_GL: 1</i>	<i>SW_GR: 3</i> <i>SW_CD: 1</i>
SW_GL	<i>SW_GL: 3</i> <i>SW_GR: 1</i>	<i>SW_GL: 3</i> <i>SW_GR: 1</i>	<i>SW_GL: 3</i> <i>ST_GR: 1</i>	<i>SW_GL: 3</i> <i>ST_GR: 1</i>	<i>SW_GL: 3</i> <i>ST_GR: 1</i>
SW_CD	<i>SW_CD: 2</i> <i>ST_CD: 1^a</i> <i>ST_CH: 1</i>	<i>SW_CD: 2</i> <i>SW_GR: 1</i> <i>ST_GA: 1</i>	<i>SW_CD: 2</i> <i>ST_CD: 1</i> <i>SW_GR: 1</i>	<i>SW_CD: 1</i> <i>ST_CD: 1</i> <i>SW_CH: 1</i> <i>SW_GL: 1</i>	<i>SW_CD: 1</i> <i>ST_CD: 1</i> <i>SW_GR: 1</i> <i>SW_CH: 1</i>
ST_CH	<i>ST_CH: 2</i> <i>ST_CD: 2</i>	<i>ST_CH: 2</i> <i>ST_CD: 2</i>	<i>ST_CH: 2</i> <i>ST_CD: 2</i>	<i>ST_CH: 2</i> <i>ST_CD: 2</i>	<i>ST_CH: 2</i> <i>ST_CD: 2</i>
ST_GA	<i>ST_GA: 4</i>	<i>ST_GA: 4</i>	<i>ST_GA: 4</i>	<i>ST_GA: 4</i>	<i>ST_GA: 4</i>
ST_GR	<i>ST_GR: 4</i>	<i>ST_GR: 4</i>	<i>ST_GR: 4</i>	<i>ST_GR: 3</i> <i>SW_GL: 1</i>	<i>ST_GR: 1</i> <i>SW_GL: 3</i>
ST_GL	<i>ST_GL: 4</i>	<i>ST_GL: 4</i>	<i>ST_GL: 4</i>	<i>ST_GL: 1</i> <i>SW_GR: 3</i>	<i>ST_GR: 4</i>
ST_CD	<i>ST_CD: 4</i>	<i>ST_CD: 4</i>	<i>ST_CD: 4</i>	<i>ST_CD: 3</i> <i>SW_CH: 1</i>	<i>ST_CD: 4</i>
Correct Rate	80%	77.5%	75%	62.5%	60%

Correct recognition is *italic*

^a This recognition was considered right because the gestures have the same meaning.

C. Recognition Results on Each Marker

In the experiment, the positions five markers on the hand were measured. However, it is possible that not all the

positions of these markers may have a high relevance to the gestures. Reducing the number of markers considered in the recognition might not only reduce the calculation but also improve the recognition accuracy. Therefore, the recognition based on each marker's motion data was calculated. Specifically, when calculating the difference between two gestures using Eq.(14) of the proposed method, only one marker was considered.

The calculation results are shown in Table III. High correct recognition rate was obtained on markers M1, M2 and M3 even that only motion data from one marker was used. The correct recognition rate was lower on markers M4 and M5, which indicates that positions of the fingers are more important in hand gestures than positions of the wrist.

In the calculation of this paper, parameter p , the number of the column to be replaced is set to be 5, which means that the last column of M was replaced to form M^* . In fact the calculation results might be different according to p . The recognition accuracy might be improved if the average or minimum difference between M and M^* when p is from 1 to n . This will be discussed in our future work.

IV. CONCLUSIONS

In this paper, a novel 3D motion analysis algorithm using SVD is proposed for gesture recognition. A gesture recognition experiment was carried out using the proposed algorithm. The experiment results verified the effectiveness of the algorithm in motion analysis and gesture recognition. The proposed algorithm is applicable for the guide robot to recognize human gestures. The algorithm will be improved and more recognition experiments will be carried in our future work.

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REFERENCES

- [1] A. Kendon, *Gesture: Visible Action as Utterance*. Cambridge University Press, Cambridge, 2004.
- [2] A.L. Hubbard, S.M. Wilson, D.E. Callan, and M. Dapretto, "Giving speech a hand: gesture modulates activity in auditory cortex during speech perception," *Human Brain Mapping*, vol.30, no.3, pp. 1028-1037, 2009.
- [3] Y.Mizobuchi, S. Wang, K. Kawata, and M. Yamamoto, "Trajectory planning method of guide robots for a achieving the guidance," *Proc. of 2005 IEEE International Conference Robotics and Biomimetics (ROBIO)*, pp.705-708, 2005.
- [4] D. Tsuda, S. Wang, N. Miura, and Y Jiang, "Development and safety validation of lead for a guidance robot," *Journal of Biomedical Fuzzy Systems Association*, vol.11, no.1, pp. 43-48 2009 [in Japanese].
- [5] S. Mitra and T. Acharya, "Gesture recognition: a survey," *IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews*, vol.37, issue 3, pp.311 - 324, 2007.
- [6] M.E. Wall, A. Rechtsteiner, and L.M. Rocha. "Singular value decomposition and principal component analysis". in *A Practical Approach to Microarray Data Analysis*. D.P. Berrar, W. Dubitzky, M. Granzow, eds. pp. 91-109, Kluwer: Norwell, MA (2003).
- [7] G.W. Stewart, "On the early history of the singular value decomposition," *SIAM Review*, vol.35, no.4, pp.551-566, 1993.
- [8] T. Ide and K. Inoue, "Knowledge discovery from heterogeneous dynamic systems using change-point correlations," *Proc. 2005 SIAM International Conference on Data Mining (SDM 05)*, pp. 571-576, 2005.
- [9] K. Mishima, S. Kanata, H. Nakanishi, Y. Horiguchi, and T. Sawaragi, "Extraction of similarities and differences in human behavior using singular value decomposition" *Proc of the 53rd annual conference of the Institute of Systems, Control and Information Engineers (SCI'09)*, pp.409-410, 2009[in Japanese].