Singular Value Analysis through Divided Time-Series Data and its Application to Walking Difficulty Evaluation

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***Abstract*—Motion time-series data observed with various sensing systems are usually analyzed to extract embodied knowledge which is remembered by the human body and reflected by the dexterity in the motion of the body. A method based on singular value analysis through divided time-series data (SVA-DTS) is proposed for extracting features from time-series data. Matrices are composed from the subsets of time-series data and the left singular vectors of the matrices are extracted as the patterns of the motion and the singular values as a scalar, by which the corresponding left singular vectors affects the matrices. The SVA-DTS was applied to a walking difficulty evaluation experiment in which three levels of walking difficulty were simulated by restricting the right knee joint. The accelerations of the middles of the shanks and the back of the waist were measured. Singular values were calculated from the normalized acceleration time-series data with the SVA-DTS. The results showed that the first singular values inferred from the acceleration data of the right shank significantly related to the increase of the restriction to the right knee. The first singular values of the acceleration data of the right shank were suggested to be reliable criteria to evaluate walking difficulty. We visualize the first singular values in a 3D space to provide intuitive information about walking difficulty which can be used as a tool for evaluating walking difficulty.**

***Keywords*—singular value decomposition (SVD), embodied knowledge, motion analysis, walking difficulty evaluation, .**

I. Introduction

The knowledge remembered by the human body and reflected by the dexterity of body motion is called embodied knowledge. Although embodied knowledge is a native human endowment that is usually not consciously accessible, it plays an important role in acquiring skills. Embodied knowledge has been studied from the viewpoint of not only neurophysiology, but also the structure of the body. Miall et al. proposed a forward model to output a signal to a controlled object via feedback control using the Smith predictor having no time delay for the control [1]. Kawato proposed a cerebellar computational model, called an internal model, which argued that the inverse model with feedback and feedforward control is useful in modeling motor control [2].

The information of motion, such as trajectory, speed, and acceleration, has been measured to analyze and extract embodied knowledge [3]–[8]. Time-series data analysis is usually necessary to extract features from the measurement data. Various methods for analyzing physical movement have been proposed. Mitra et al. [9] surveyed methods of gesture recognition and suggested that the Hidden Markov model is effective for gesture motion data analysis. However, the Hidden Markov model is not effective when the number of states is large or the data is discontinuous. Lamar et al. [10] proposed a neural network, Temporal-CombNET, and applied it to Japanese-Kana finger spelling recognition. Since a neural network is too sensitive in time-series data length, the accuracy is not very good. Jerde et al. [11] measured the angles of hand joints for recognizing fingerspelling hand shapes, and reduced the dimensions of the hand by discriminant analysis by Principal Component Analysis (PCA). Daffertshofer et al. [12] also suggested the effectiveness of PCA for reducing high-dimensional time-series data sets to a small number of modes in the analysis of gait kinematics. PCA reduces the number of explanatory variables and is a model for visualization with principal component variables. It is possible, however, to lose significant principal component variables when the proportion of variance is low and the number of data is inadequate. In other words, the accuracy of PCA declines when the contribution ratio is low due to a shortage of data.

In previous studies, authors proposed a method based on singular value decomposition (SVD) to extract features from time-series data [13], [14]. The left and right singular vectors and the singular values are decomposed from a Hankel matrix defined from the time-series data, which are measured with sensors. Since the left singular vector represents the characteristics of the Hankel matrix, and the singular value represents the strength of the corresponding left singular vector, SVD is used generally as a method for extracting characteristics from observed time-series data. The proposed method was applied to a hand gesture recognition experiment and the experiment results confirmed the usefulness of SVD. However, the proposed method requires complicated preprocessing, such as segmentation, since the matrix is composed of time-series data from each cycle of the motion, which also make it not applicable in real-time analysis.

In this paper, we propose a new method, to simplify preprocessing, to extract features based on singular value analysis through divided time-series data (SVA-DTS). Section II describes the SVA-DTS. Section III validates the SVA-DTS with a walking difficulty evaluation experiment. Three levels of walking difficulty were simulated and the singular values extracted from the acceleration data during are used as a criterion to evaluate walking difficulty. Finally, section VI concludes the paper and discusses the future work.

II. Embodied Knowledge Extraction Using SVA-DTS

SVD is a factorization of a rectangular real or complex matrix with many applications in signal processing and statistics [15], [16]. Suppose M is an m-by-n matrix, then a factorization of M is M=$U\Sigma V$, where U=(u_1, u_2, ..., u_m) contains the left singular vectors of M, V=(v_1, v_2, ..., v_n) contains the right singular vectors of M, and the matrix Σ is an m-by-n diagonal matrix with nonnegative real singular values on the diagonal. Recently, SVD has been used in time-series data analyses for data mining [17] and motion analyses to study the coordinative structures in human behavior [18].

Suppose that w points $(P_1, P_2, \cdots, P_w)$ of the body are measured while a person is performing a motion. The time series data on each point are divided into K subsets. On point P_i, the measured data series of the G-th subset is denoted as $\tau^{i,G}$. The data series of $\tau^{i,G}$ consists of three-dimensional data $(X^{i,G}, Y^{i,G}, Z^{i,G})$. From this time-series data $\tau^{i,G} = (X^{i,G}, Y^{i,G}, Z^{i,G})$, n vectors by m data sampling are extracted by overlapping and the matrices $M_X^{i,G}$, $M_Y^{i,G}$, and $M_Z^{i,G}$ are constructed as a collective of the measurement data on the X, Y, and Z coordinates of the motion, respectively. Fig. 1 shows a design for constructing the Hankel matrix $M_X^{i,G}$. The matrices $M_X^{i,G}$, $M_Y^{i,G}$, and $M_Z^{i,G}$ are described as follows:

$$M_X^{i,G} = (X_1^{i,G}, X_2^{i,G}, \cdots, X_n^{i,G})^T \quad (1)$$
$$M_Y^{i,G} = (Y_1^{i,G}, Y_2^{i,G}, \cdots, Y_n^{i,G})^T \quad (2)$$
$$M_Z^{i,G} = (Z_1^{i,G}, Z_2^{i,G}, \cdots, Z_n^{i,G})^T \quad (3)$$

where $X_p^{i,G} = (x_{p,1}^{i,G}, x_{p,2}^{i,G}, \cdots, x_{p,m}^{i,G})$, $p = 1, 2, \cdots, n$, and x is a datum on the X coordinate. $Y_p^{i,G}$ and $Z_p^{i,G}$ are defined in the same way. Suppose $M_k^{i,G}$, $k = \{X, Y, Z\}$ is an m-by-n matrix as the general format of $M_X^{i,G}, M_Y^{i,G}, M_Z^{i,G}$. The SVD of the matrix $M_k^{i,G}$ is

$$M_k^{i,G} = U_k^{i,G} \Sigma_k^{i,G} \{V_k^{i,G}\}^T \quad (4)$$

where $U_k^{i,G} = (u_{1,k}^{i,G}, u_{2,k}^{i,G}, \cdots, u_{m,k}^{i,G})$ is an m-by-m unitary matrix, $\{V_k^{i,G}\}^T$ denotes the conjugate transpose of $V_k^{i,G} = (v_{1,k}^{i,G}, v_{2,k}^{i,G}, \cdots, v_{n,k}^{i,G})$ which is an n-by-n unitary matrix, and the matrix $\Sigma_k^{i,G}$ is an m-by-n diagonal matrix. The diagonal entries of $\Sigma_k^{i,G}$ are the singular values of $M_k^{i,G}$. The matrix $U_k^{i,G}$ contains the left singular vectors of $M_k^{i,G}$ and the matrix $V_k^{i,G}$ contains the right singular vectors of $M_k^{i,G}$. The left singular vector in $U_k^{i,G}$ expresses the characteristic of the whole time-series data better if its corresponding singular value in $\Sigma_k^{i,G}$ is larger. That is, the greater the singular value is, the more dominant the corresponding pattern is.

Suppose that the number of left singular vectors to be considered is l, and the element number of the j th left singular vector is q. Let us denote the couples of the singular values and the left singular vector as

$$H_G = ((\sigma_{1,X}^{i,G}, u_{1,X}^{i,G}), (\sigma_{2,X}^{i,G}, u_{2,X}^{i,G}), \cdots, (\sigma_{l,X}^{i,G}, u_{l,X}^{i,G})) \quad (5)$$

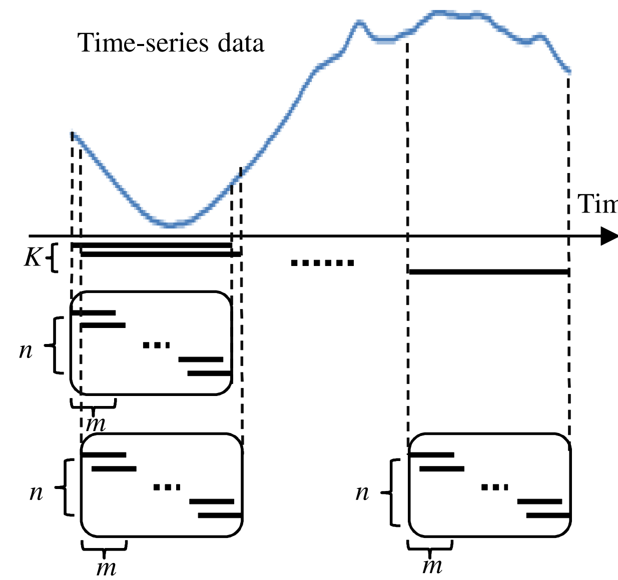

Fig. 1: Design of Matrix $M_X^{i,G}$ in SVA-DTS

for $u_{j,X}^{i,G} = (\hat{u}_{1j,X}^{i,G}, \hat{u}_{2j,X}^{i,G}, \cdots, \hat{u}_{hj,X}^{i,G}, \cdots, \hat{u}_{qj,X}^{i,G})$ in the descending order of the singular values, where $\hat{u}_{hj,X}^{i,G}$ is the h th element of the j th left singular vector $u_{j,X}^{i,G}$. $\sigma_{1,k}^{i,1}$ of H_1, for example, is the first (and the biggest) singular value of the first subset from the k coordinate of time-series data measured on measurement point i. The pair $(\sigma_{1,X}^{i,G}, u_{1,X}^{i,G})$ possesses the most important feature of the G-th subset

Since the matrix is designed by overlapping the extracted data from the whole time-series data, preprocessing, such as segmentation, is not required. The SVA-DTS is less constrained by the length of the whole data than those using a neural network. The difference between PCA and our method is that PCA generally analyzes the matrix composed of the deviation of each datum from the empirical mean and does not allow overlap of the measurement data.

III. Walking Difficulty Evaluation

A. Acceleration Measurement with Wearable Wireless Accelerometers

Accelerometers have been widely used to monitor body movements, including gait, sit-to-stand transfers, postural sway, and falls because they are compact, wearable, and convenient to use [19]–[22]. In the experiment, we examined the acceleration of walking difficulty simulated by restricting the right knee joint with knee supporters and weight bands (Fig. 2). The knee supporter bound around the knee joint decreases the range of movement (ROM) of the knee joint and the weight band bound around the ankle joint can simulate the weakness in muscle strength. The simulation is very important in testing our method since it does not endanger the safety of the disabled during the development phase of the method. Two levels of walking difficulty were simulated by two levels of constraint in the experiment. The weak constraint was simulated with one knee supporter and one weight band (1 kg), and the strong constraint with two knee supporters and two weight bands (2 kg). In total, three statuses (Normal without constraint, Weak, and Strong) were examined. Six healthy volunteers (YJ, TK,

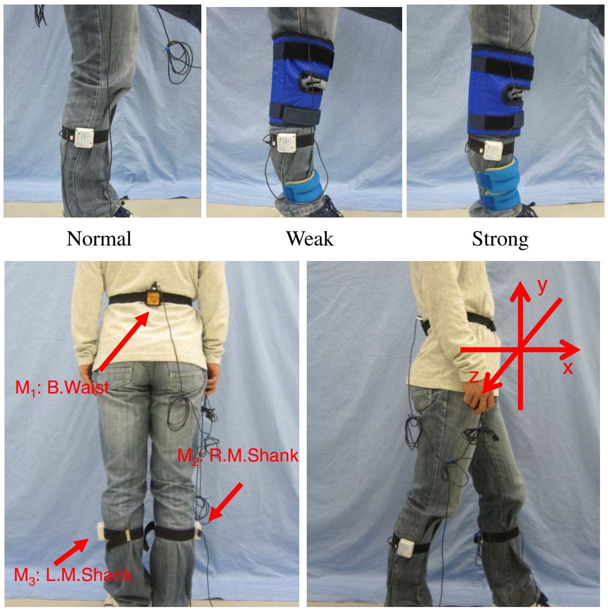

Fig. 2: Experiment Environment for Ambulation

KT, KS, TF, and RT; 5 males and 1 female) aged 21-31 yr (mean 26 yr) participated in the experiment. The subjects were instructed to walk for approximately 4 m along a straight line. The experiment was carried out in the status order of Normal, Weak, and Strong. For each status, each subject walked four times.

The acceleration is measured by 3 wearable wireless 3-axis accelerometers (Motion Recorder MVP-RF8, Microstone Nagano, Japan): P1 on the back of the waist (B. Waist), P2 on the midpoint of the right shank (R. M. Shank), and P3 on the midpoint of the left shank (L. M. Shank), as shown in Fig. 2. Sampling rate of the sensor was 100 Hz. When the subject stands upright, the sensors' x-axis is front/back, y-axis up/down, and z-axis right/left. However, since the orientation of the sensors changes during walking, the coordination system will also change.

The measured time-series data in the x coordinate of the three sensors when subject YJ walked in the status of Weak are shown in Fig. 3. The fluctuation of the acceleration was significant when the right foot or the left foot was landing. Fig. 3a shows five strides. The acceleration data of the first stride were extracted and are shown in Fig. 3b. All the acceleration at the B. Waist, R. M. Shank, and L. M. Shank significantly fluctuated when the right foot pushed off from the floor or stepped on the floor. The fluctuation in the acceleration at L. M. Shank was more significant than that at R. M. Shank since the right leg was restricted by the knee supporter and the weight band. The fluctuation in the acceleration at B. Waist was the smallest among the three measurement points. This showed that the trunk of the body, especially the waist, was kept relatively stable to maintain the body balance even when the lower limbs were disabled.

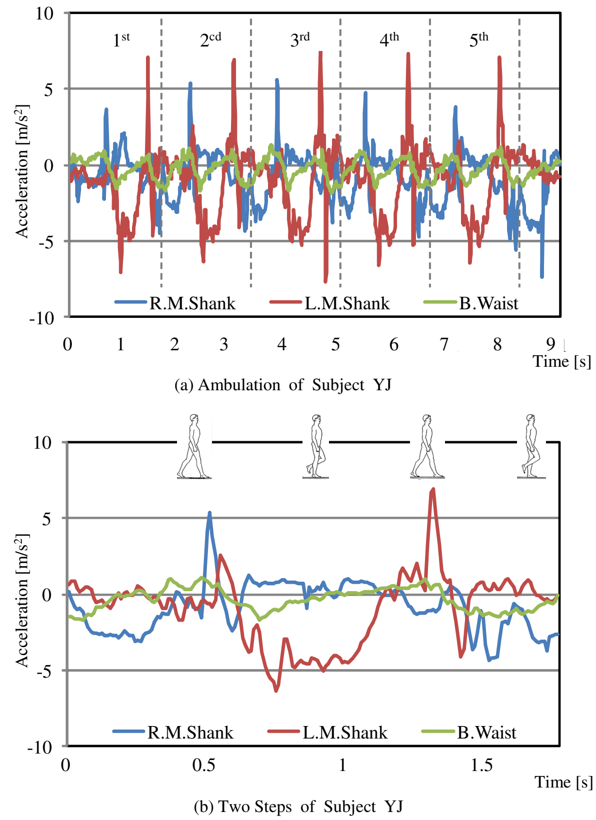

Fig. 3: Example of Ambulation

B. Data Analysis Using SVA-DTS

Since the subject walked slower with stricter constraint, the maximum and minimum accelerations were affected by the constraint. Consequently, the effectiveness of the SVA-DTS cannot be validated if the SVA-DTS is performed on the raw data because the values of the singular values are affected by the values of the items in the Hankel matrix. Therefore, the time series data are normalized in order to exclude the impact of the values of the acceleration. For every series of the acceleration data, the maximum acceleration a_{max} is set to 1, the minimum acceleration a_{min} to 0, and the other values $a_{nl} = (a - a_{min})/(a_{max} - a_{min})$.

After normalization, the SVA-DTS is performed on every series of acceleration data. Hankel matrices with 10 rows and 10 columns were composed from the subsets of data, that is $n = 10$, $m = 10$. The number of subsets K was determined by the total number of data in each acceleration time-series data N ($K = N - m - n + 1$). Only the first singular value and the first left singular vector were considered in the current analysis. Thus parameter l was 1. The first singular values of all the H_Gs of a single series of acceleration were sorted in descending order and the top 50 first singular values were adopted as the criterion to evaluate the walking difficulty. There were 3 measurement points P_1, P_2, P_3, each of which had 3 dimensions. Therefore, 9 groups of first singular values were calculated for each subject.

TABLE I: Singular Values

Subjects	Constraint Ambulation	P_1 S_x	P_1 S_y	P_1 S_z	P_2 S_x	P_2 S_y	P_2 S_z	P_3 S_x	P_3 S_y	P_3 S_z
TF	Normal	6.66	6.75	5.01	5.15	3.70	5.97	4.35	8.03	5.76
	Weak	7.87	7.57	4.89	5.05	3.25	4.94	4.67	7.81	5.27
	Strong	8.19	7.89	4.60	4.33	4.59	3.96	6.58	7.81	4.94
YJ	Normal	8.17	8.43	7.21	6.47	4.65	7.90	6.50	6.78	7.84
	Weak	8.59	8.81	6.82	7.28	6.00	7.12	6.57	7.17	7.72
	Strong	8.72	9.13	6.63	7.62	4.74	5.13	6.37	6.94	8.01
TK	Normal	8.08	7.31	6.95	5.38	4.83	8.33	5.91	6.19	8.22
	Weak	8.66	7.78	7.35	6.92	5.77	7.73	6.10	6.88	7.25
	Strong	8.92	7.86	5.77	7.10	6.12	5.60	6.85	6.18	7.81
KS	Normal	8.52	7.11	5.70	4.83	3.74	7.02	6.98	7.22	7.26
	Weak	8.06	7.66	5.77	5.57	4.32	5.39	6.64	7.72	5.72
	Strong	8.62	7.11	5.24	5.50	4.58	4.54	5.91	7.34	5.34
RT	Normal	8.07	6.66	6.36	6.03	5.53	6.99	5.89	7.26	6.01
	Weak	8.45	6.49	5.58	7.19	6.98	5.13	5.41	7.17	6.46
	Strong	8.02	6.33	5.83	8.33	6.32	4.31	5.51	6.95	7.06
KT	Normal	7.94	7.96	4.53	5.97	4.69	6.39	5.06	6.74	6.70
	Weak	8.11	7.25	4.35	6.57	4.82	4.08	4.62	6.01	6.23
	Strong	8.19	7.03	4.47	7.94	6.24	4.59	5.40	4.60	6.04
Ave.	Normal	7.91	7.37	5.96	5.64	4.52	7.10	5.78	7.04	6.97
	Weak	8.29	7.59	5.79	6.43	5.19	5.73	5.67	7.13	6.44
	Strong	8.44	7.56	5.42	6.80	5.43	4.69	6.10	6.64	6.53

C. Results and Discussions

The mean values of the top 50 first singular values extracted from the acceleration data calculated with the SVA-DTS are listed in Table I. On measurement point P_1 that was located at B. Waist, no common changing trend was found with the 3 levels of walking difficulty (Normal, Weak, and Strong). The waist is the center of the body and is always kept balanced during walking. Fig. 3 shows that the waist was relatively stable to maintain the body balance. On measurement point P_3 that was located at L. M. Shank, although increasing or decreasing trends were found with the walking difficulty in some subjects, there were no consistent trends among the subjects. The constraint to the right knee also affected the gait of the left leg. However, the effect was different among the subjects. On measurement point P_2 that was located at R. M. Shank, common changing trends were observed in the mean values with the increase of walking difficulty. On the x and y coordinates, the singular values increased with the increasing walking difficulty. On the z coordinate, the singular values decreased with the increasing walking difficulty. Since P_2 was located on the same side of the restricted knee, the acceleration measured by P_2 most reflected the walking difficulty posed by the constraint.

A one way analysis of variance (ANOVA) was performed on the singular values at P_2 to assess the significance of the difference among the three levels. The results in Table II show that for the z coordinate, the singular values are significantly different between the three levels. The first singular values of P_2, therefore, are suggested to be an effective criterion to evaluate walking difficulty. That is, the first singular values from the z coordinate acceleration of the restricted leg decreases with the increase of walking difficulty. The larger the first singular values are, the more serious the walking disability is. The results in Table I and Table II show the possibility of evaluating walking disability using the singular values. In order to provide a more understandable presentation of the data for the physical therapists and the patients, we propose a visualization tool to assist in the evaluation of walking difficulty. The first singular values extracted from the acceleration data at P_2 are plotted in a 3D space in Fig. 4. The singular values of the 4 times of walking are plotted in different shape and color according to the 3 statuses. The average singular values of the 3 statuses are connected by black lines. Fig. 4 shows that the first singular values are clustered according to the statuses. The line connecting the average values can be considered as severity line of walking difficulty. The visualization tool can be utilized in diagnosis or rehabilitation to assist walking difficulty evaluation.

TABLE II: Results of ANOVA on P_2

Variation Factor		Sum of Sq. Diff.	DOF	Mean Sq.	F Val.	P Val.
S_x	Be.Gr.	4.247	2	2.123	1.739	0.209
	With.Gr.	18.311	15	1.221		
	Total	22.558	17			
S_y	Be.Gr.	2.656	2	1.328	1.317	0.297
	With.Gr.	15.125	15	1.008		
	Total	17.781	17			
S_z	Be.Gr.	17.554	2	8.777	8.528	0.003
	With.Gr.	15.438	15	1.029		
	Total	32.992	17			

Although statistically not significant, the singular values on

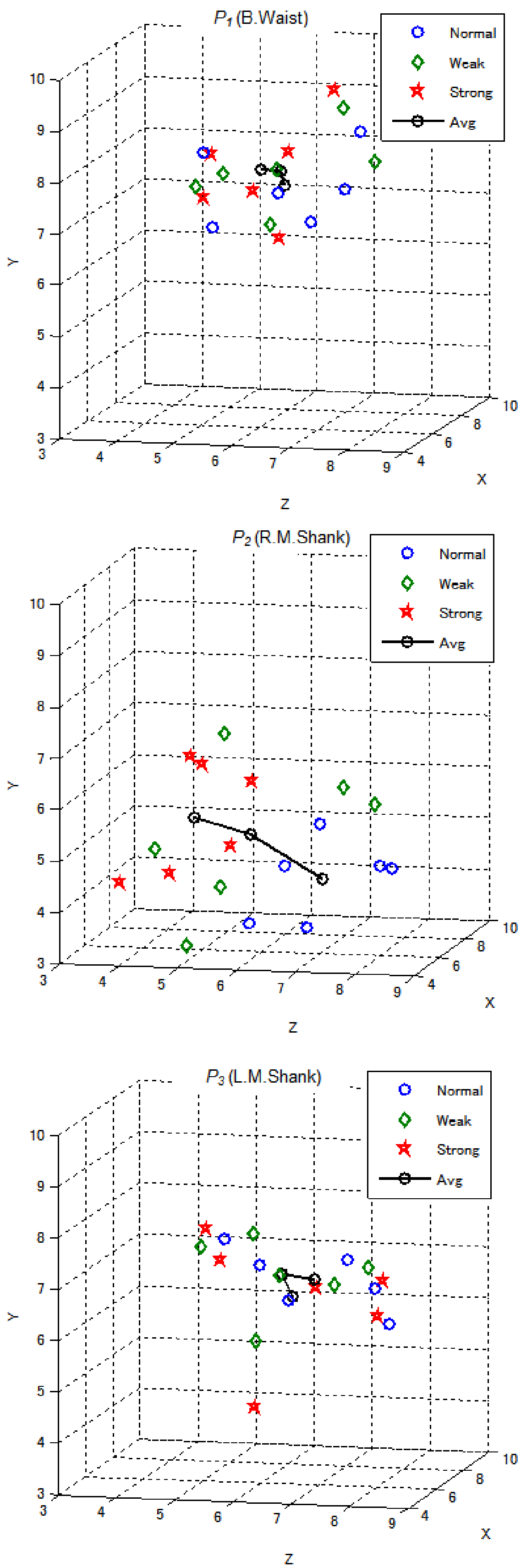

Fig. 4: Singular Values Plotted in 3D Spaces

TABLE III: Difference between Singular Values at P_2

		$S_z - S_x$	$S_z - S_y$			$S_z - S_x$	$S_z - S_y$
TF	N[a]	0.82	2.26	KS	N	2.18	3.28
	W	-0.11	1.69		W	-0.18	1.07
	S	-0.36	-0.63		S	-0.96	-0.04
YJ	N	1.43	3.26	RT	N	0.96	1.46
	W	-0.16	1.13		W	-2.06	-1.86
	S	-2.49	0.39		S	-4.02	-2.01
TK	N	2.95	3.50	KT	N	0.42	1.70
	W	0.81	1.96		W	-2.49	-0.75
	S	-1.50	-0.52		S	-3.35	-1.65

[a]N: Normal, W: Weak, S: Strong

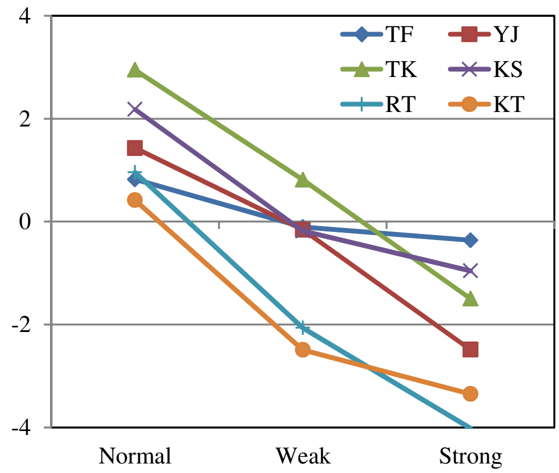

Fig. 5: $S_z - S_x$ at P_2

the x and y coordinates at P_2 increased on average with the increasing walking difficulty. The differences of the singular values between different coordinates, $S_z - S_x$ and $S_z - S_y$, were both in descending trends, as shown in Table III. The $S_z - S_x$ and $S_z - S_y$ plotted in Fig. 5 and Fig. 6 for each subject illustrate descending trends with the increasing walking difficulty without exception. The differences is suggested to be effective in evaluating walking difficulty. On the other hand, a great inter-subject difference was also observed, which was caused by the difference in the dynamics of human walking. These results suggest that the $S_z - S_x$ and $S_z - S_y$ are effective in monitoring a person's walking difficulty(e.g., tracking the recovery of a patient during rehabilitation) while the inter-subject difference cannot be neglected when they are applied to evaluate walking difficulty between different persons.

Since the acceleration time-series data were normalized before performing the SVA-DTS, the analysis results above validate that the proposed SVA-DTS is effective to extract features from time-series data. With the method in the previous study [14], the knee joint restricted was not able to be identified since both the singular values at P_2 and P_3 significantly decreased with the increasing walking difficulty. By contrast, the singular values calculated with the SVA-DTS only significantly decreased with the increasing walking difficulty at P_2 on the restricted right leg. This suggests that the SVA-DTS is more effective in extracting features from time-series data than the previous method.

The singular values indicate the importance of their corre-

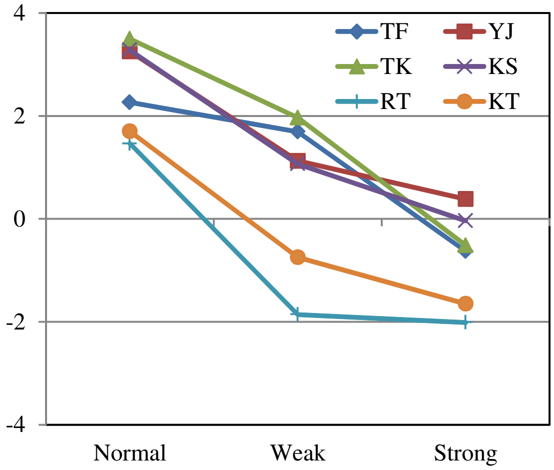

Fig. 6: $S_z - S_y$ at P_2

sponding left singular vectors that express the characteristics of the time-series data. Only the first singular values were considered in the current analysis to evaluate the walking difficulty. The detailed gait pattern change with walking difficulty can be extracted by the left singular vectors. The usefulness of the left singular vectors will be studied in our future study.

IV. Conclusion

The SVA-DTS for embodied knowledge extraction from the time-series data of motion is proposed based on the SVD. The effectiveness of the SVA-DTS was shown by the walking disability evaluation using the singular values. The analysis results show the possibility to evaluate walking disability using the first singular values. The results of the experiment suggest that the SVA-DTS is effective for extracting embodied knowledge from the time-series data. The visualization tool plotting the singular values in a 3D space was proposed to assist in the evaluation of walking difficulty. Future work will study the usefulness of the left singular vectors and more experiments will be conducted to further validate the SVA-DTS.

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